JMO Sequence Questions

Puzz Points: 11

Level: Junior Ref No: J26

[JMO 2001 B2] In a sequence, each term after the first is the sum of the squares of the digits of the previous term. Thus if the first term were 12, the second term would be $1^2 + 2^2 = 5$, the third term $5^2 = 25$, the fourth term $2^2 + 5^2 = 29$, and so on.

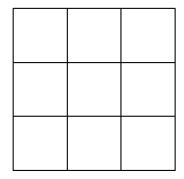
- (i) Find the first five terms of the sequence whose first term is 25.
- (ii) Find the 2001st term of the sequence whose first term is 25.

Solution: (i) 25, 29, 85, 89, 145 (ii) 16

Level: Junior Ref No: J32 Puzz Points: 11

[JMO 2011 B3] A 3 \times 3 grid contains nine numbers, not necessarily integers, one in each cell. Each number is doubled to obtain the number on its immediate right and trebled to obtain the number immediately below it.

If the sum of the nine numbers is 13, what is the value of the number in the central cell?



Solution: $\frac{6}{7}$

Level: Junior Ref No: J42 Puzz Points: 15

[JMO 2008 B6] In a sequence of positive integers, each term is larger than the previous term. Also, after the first two terms, each term is the sum of the previous two terms.

The eighth term of the sequence is 390. What is the ninth term?

Solution: 631

Level: Junior Ref No: J49 Puzz Points: 10

[JMO 2010 B1] In a sequence of six numbers, every term after the second term is the sum of the previous two terms. Also, the last term is four times the first term, and the sum of all six terms is 13.

What is the first term?

Solution: $1\frac{1}{4}$

Level: Junior Ref No: J61 Puzz Points: 10

[JMO 2005 B1] The first three terms of a sequence are $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$. The fourth term is $\frac{1}{2} - \frac{1}{3} + \frac{1}{4}$; henceforth, each new term is calculated by taking the previous term, subtracting the term before that, and then adding the term before that.

- (i) Write down the first six terms of the sequence, giving your answers as simplified fractions.
- (ii) Find the 10th term and the 100th term, and explain why they have to be what you claim.

Solution: (i)
$$\frac{1}{4}$$
, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{5}{12}$, $\frac{1}{4}$, $\frac{1}{3}$ (ii) 10^{th} term is $\frac{1}{3}$, 100^{th} term is $\frac{5}{12}$

Level: Junior Ref No: J63 Puzz Points: 12

[JMO 2005 B3] The diagram below is to be completed so that each box contains a whole number, the total of the numbers in the thirteen boxes is 2005 and the sum of the numbers in any three consecutive boxes is always the same.

In how many different ways is it possible to complete the diagram in this way?

	175						70	

Solution: One way (the first term is 205 only)